Cellular automata model simulating complex spatiotemporal structure of wide jams

Xiao-Bai Li, Rui Jiang, and Qing-Song Wu*
School of Engineering Science, University of Science and Technology of China, Anhui, Hefei, 230026, People's Republic of China
(Received 2 January 2003; published 21 July 2003)

According to the empirical observation of highway traffic, inside wide moving jams there is a complex spatiotemporal structure: jam is not compact and relatively large values of the time and the distance headway are visible. We present a cellular automata model by introducing ''jam headway'' and ''jammed status'' to simulate that complex structure. Using computer simulations, the fundamental diagram, the space-time plots, the time series of the density in the jams, and the 1-min average data of this model are analyzed. It is shown that compared to other existing models, this model can display the experimental characteristics of the wide moving jams.

DOI: 10.1103/PhysRevE.68.016117 PACS number(s): 89.40.−a, 64.60.Cn, 02.60.Cb, 05.70.Ln

I. INTRODUCTION

In the last few decades, traffic problems have attracted the interests of many scientists. Numerous experimental investigations of traffic have been made to understand phenomena of traffic flow (see, e.g., Refs. [1,2]). According to the analysis of real measurements, three traffic phases, namely, (1) free flow, (2) synchronized traffic, and (3) wide moving jams are distinguished. Free flow traffic is identified by a large average velocity at low density. In synchronized traffic, the velocity is considerably smaller than that in free flow but the flow can still have large values. In wide moving jams, one observes a high density and negligible average velocity and flow.

To understand the behavior of traffic flow, various traffic flow models have been proposed and studied, including car-following models, cellular automata (CA) models, gas-kinetic models, and hydrodynamic models (see Ref. [5] for an overview). Compared to other dynamical approaches, CA models are conceptually simpler, and can be easily implemented on computers for numerical investigations [6]. So, it developed very quickly in the last decade after the first CA model was proposed in 1992 by Nagel and Schreckenberg (NS model) [7]. The NS model is an essential and minimal CA model for highway traffic flow. Although it has simple rules, the model is able to represent the basic phenomena observed in real traffic such as the start-stop waves in high density flow. Nevertheless, for the description of more complex situations, such as the metastable state and the hysteresis phenomenon, more detailed rules have to be added to the basic model. Slow-to-start (STS) rules, which try to model the restart behavior of standing vehicles, is now widely accepted as an important ingredient for the occurrence of metastable states [8,9]. Velocity effect (VE) model, considering the movement of preceding vehicle, gives a more realistic fundamental diagram [10].

Let us look at some detailed phenomena of different phases of real traffic. According to the experimental results [3], inside the free flow and wide moving jams there are complex structures. That is, in free flow platoon of vehicles driving bumper to bumper with a time headway well below 1 s occurs; in wide moving jams there can still exist relatively large gaps, which indicate that the wide jams are not compact in the sense that all the cars are piled up bumper to bumper. The former can be explained by the thought of VE model: individual drivers will take the preceding car as a moving object instead of an immobile obstacle, especially when they both have large velocities. As a result, we can represent that vehicles platoon with large velocities by VE model (see Sec. II).

In this paper, we focus on the complex structure of wide jams. It has been well known that a wide jam has two characteristics, the upstream moving velocity (about -15 km/h) and the outflow from the jam (about 1800 vehicles/h see, e.g., Refs. [3,11]). However, the field data in Ref. [3] also reveal the existence of complex structure inside the wide jams. In fact, there are findings in 1996 [4], saying that *inside traffic jams was a complex space-time structure of the flux: the flux of vehicles could be changed from zero (a standstill) to some values*. This phenomenon is explained as follows. When an individual driver notices that the cars in front of him are slowing down or standstill, he will brake in advance even if there is still large headway between the two successive cars. This maneuver causes blanks between vehicles inside the traffic jam and drivers who reduce those blanks will induce new upstream blanks. Then the complex spatiotemporal structure forms and gives fluctuant flux inside wide moving jams. We can imagine the practical significance of this problem, since it triggers the ''stop-and-go'' phenomena inside the wide jams and concerns the exhaust emission of those jammed vehicles.

Nevertheless, our simulations show that in the VE model combined with STS rules, such wide jams with complex structure as well as the two characteristics cannot be reproduced, all the cars in the jams are piled up bumper to bumper (see Sec. II). This is in contradiction with the empirical observations. Thus, in this paper, we try to deliver a better understanding of the jams by presenting a CA model in which the concepts of ''jam headway'' and ''jammed status'' are introduced. It is shown that the model can exhibit such complex spatiotemporal structure as well as the two characteristics of jams, therefore shedding some light on the influence of driver reaction on traffic flow state. In the following

*Corresponding author. Email address: qswu@ustc.edu.cn

 ©2003 The American Physical Society

section, the model is presented and the simulation results are analyzed. The conclusions are given in Sec. III.

II. MODEL AND SIMULATION

In this section we propose a model for the description of the complex structure of wide jams. We introduce ‘‘jammed status’’ with ‘‘jammed maximum velocity’’ M_{jam} (smaller than the maximum velocity M) as a characteristic of it and ‘‘jam headway’’ d_{jam} as a criterion distinguishing the status of vehicle. When the gap between a vehicle and its proceeding vehicle is no larger than the jam headway and the velocity of the proceeding vehicle is no larger than the jammed maximum velocity, the following vehicle will fall into the jammed status. As a result, the maximum velocity of it is reduced to M_{jam}.

The whole update rules of the model at each discrete time step $t \rightarrow t+1$ are as follows.

Step 1: Status judging

$$V_{max}=\begin{cases} M_{jam} & \text{if } d_{i,t} \leqslant d_{jam} \text{ and } U_{i+1,t} \leqslant M_{jam} \\ M & \text{in all other cases.} \end{cases} \tag{1}$$

Step 2: Probability choosing

$$p(U)=\begin{cases} P_0 & \text{if } U_{i,t}=0 \\ P & \text{if } U_{i,t}>0. \end{cases} \tag{2}$$

Step 3: Acceleration

$$V=\min\{U_{i,t}+1, V_{max}\}. \tag{3}$$

Step 4: Braking

$$\text{if } V>d_{i,t}+V'_{i+1,t+1} \text{ then } V=d_{i,t}+V'_{i+1,t+1}. \tag{4}$$

Step 5: Randomization

$$\text{if } V>0 \text{ then } U_{i,t+1}=\max\{0, V-1\} \tag{5}$$

with probability $p(U)$.

Step 6: Moving

$$X_{i,t+1}=X_{i,t}+U_{i,t+1}. \tag{6}$$

Here $X_{i,t}$ and $U_{i,t}$ are the position and velocity of vehicle i. $d_{i,t}=X_{i+1,t}-X_{i,t}-1$ is the gap of the vehicle i (it is assumed that vehicle $i+1$ precedes vehicle i). $V'_{i+1,t+1}$ is the virtual velocity of $i+1$ car at $t+1$, defined as in VE model (cf. Ref. [10]). P_0 and P are randomization probability of standing vehicles and moving vehicles, respectively.

In the simulations, a circuit road is used. The road is divided into cells of length 1.5 m and each vehicle has a length of five cells. One time step corresponds to 1 s. The vehicles move from the left to the right with periodic boundary conditions. It is assumed that $M=25$, which corresponds to 135 km/h, just as the normal free flow speed in the real traffic. M_{jam} is assumed to be 1, corresponding to 5.4 km/h, a characteristic velocity in jammed status of real traffic. d_{jam} is 10 cells corresponding to a length of two vehicles. P_0 is assumed to be 0.5, indicating a standstill vehicle needs about

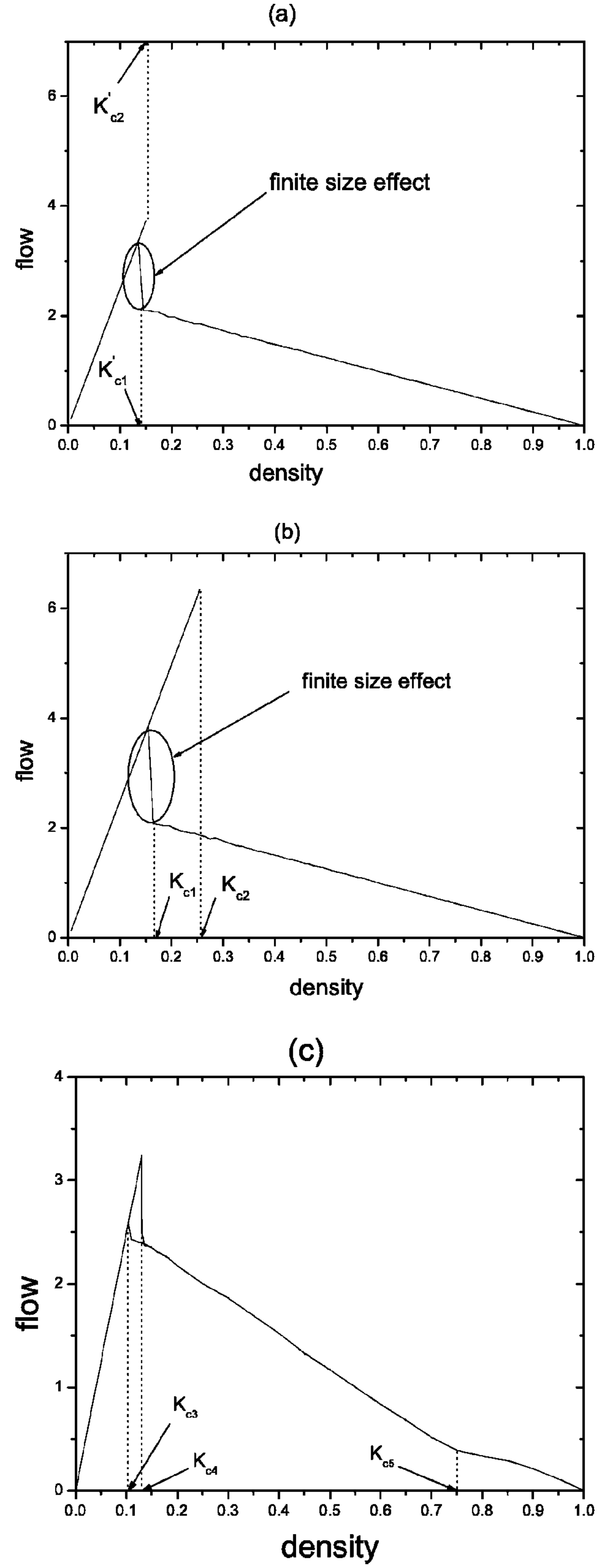

FIG. 1. The fundamental diagrams of (a) VDR model, $L=10\,000$, (b) VE-STS model, $L=10\,000$, and (c) the present model, $L=250\,000$.

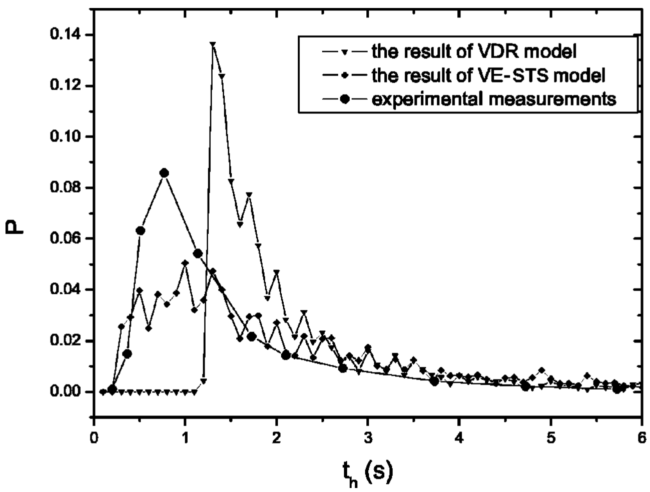

FIG. 2. The distribution of time headways in free flow in VE-STS model and VDR model.

2 s to accelerate [3]. For moving vehicles, $P = 0.1$ is used. The data are collected after the time evolution reaches to the 200 000th step.

First, let us look at the results of the model without the concepts of M_{jam} and d_{jam}, which may be viewed as the VE model combined with STS rules (VE-STS model). Note that if one lets the virtual velocity be 0, then the VE-STS model reduces to velocity-dependent randomization (VDR) model [9]. The fundamental diagram of VE-STS model under the same parameters as given above is shown in Fig. 1(b). The left branch starts from homogeneous initial distribution, where only the free flow exists. The right branch starts from a megajam, and the system is a coexistence of free flow and jam. As the result of considering the proceeding car's velocity, metastable state is between k_{c1} and k_{c2}, an obviously larger range than that of VDR model, which is between k'_{c1} and k'_{c2} [cf. Fig. 1(a)[1]].

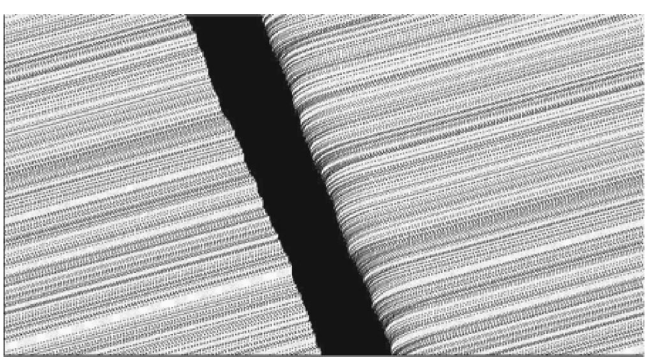

FIG. 3. The space-time plot of the coexistence of the free flow and jam in VE-STS model, where $k = 0.20$ and $L = 5000$. The cars are moving from the left to the right, and the vertical direction (up) is (increasing) time.

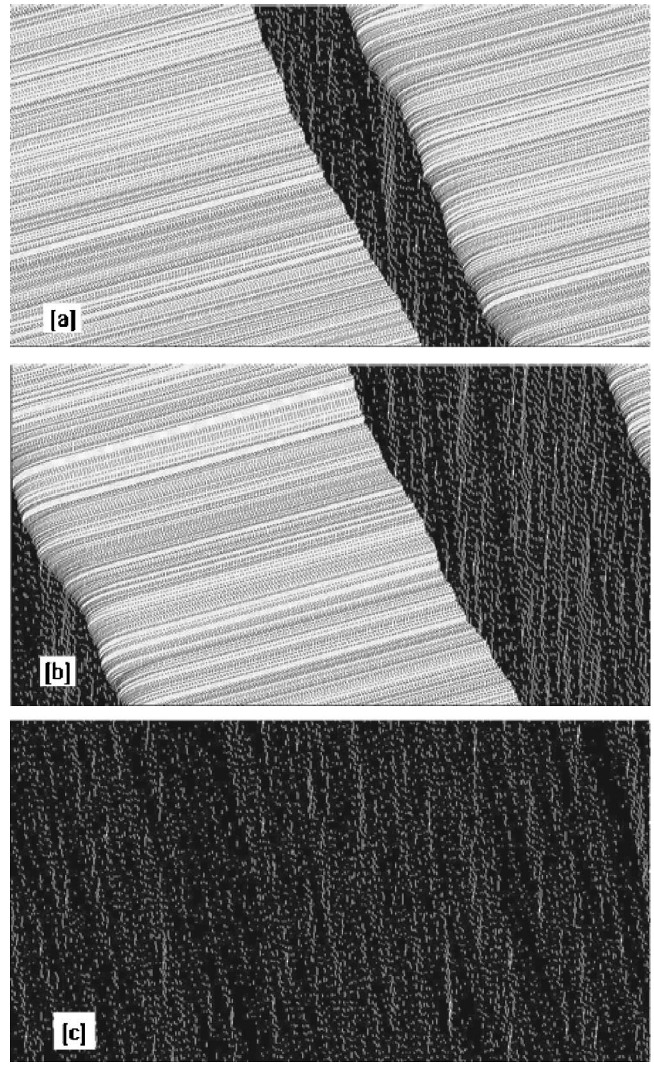

FIG. 4. The space-time plot of the present model. (a) $k = 0.20$, (b) $k = 0.35$, (c) $k = 0.80$. The system size $L = 5000$.

Figure 2 shows the distributions of the time headways of vehicles in free flow of both VE-STS model and VDR model. The experimental measurements of real free traffic are also presented for comparison (see Fig. 10 in Ref [3]). One can see that in the VDR model, the minimum time headway that can be reached is about 1.2 s, which is much larger than that obtained from field (about 0.2 s). This implies that the VDR model cannot reproduce the platoon of vehicles driving bumper to bumper with a time headway well below 1 s in free flow.

With the introduction of the velocity effect of proceeding cars, one can see that the minimum time headway decreases to approximately 0.2 s, which is in good agreement with empirical data. Moreover, the distribution profile in VE-STS model is much closer to the measurements than that in the VDR model. All these results show the importance of anticipation effects in free flow; therefore, it is necessary to include the velocity effect of proceeding cars when modeling the complex traffic phenomena.

Let us look at the structure of the jams in the VE-STS model. In Fig. 3, the space-time plot of the coexistence of

[1]In Fig. 1(a), $L = 10\,000$ is used to reproduce the metastable state, which will disappear in large system. The same system size is used in Fig. 1(b) for comparison.

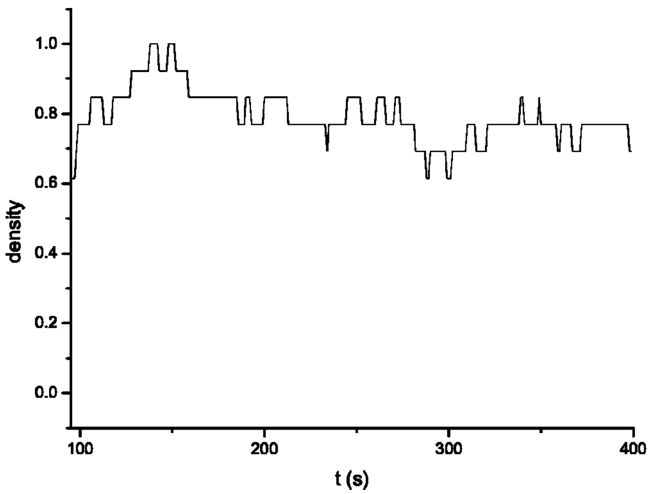

FIG. 5. The time series of the density inside the sparse jams in the present model.

free flow and jam is shown. One can see that the jam is completely compact in the VE-STS model: vehicles are standstill bumper to bumper until the downstream front of the jam moves to it.[2]

Next we present the results of the present model. In Fig. 1(c), the fundamental diagram of the model is shown. Similar to that of the VE-STS model, there are also two branches. The left branch starts from homogeneous initial distribution corresponding to free flow and the right branch starts from a megajam corresponding to congestion state. The metastable state exists between k_{c3} and k_{c4}. Nevertheless, different from the VE-STS model, the right branch of the new model can further be classified into two segments.

Let us focus on the right branch. In Fig. 4(a), we show the space-time plot of the present model at the density $k=0.2$.

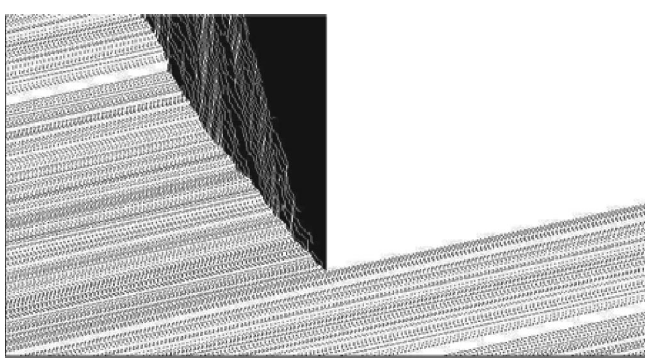

FIG. 6. The evolution of the traffic behind a traffic light. $k=0.1$. The system is run until it reaches the stationary state of free flow and then the traffic light is switched to red, i.e., vehicles are forbidden to pass the signal site.

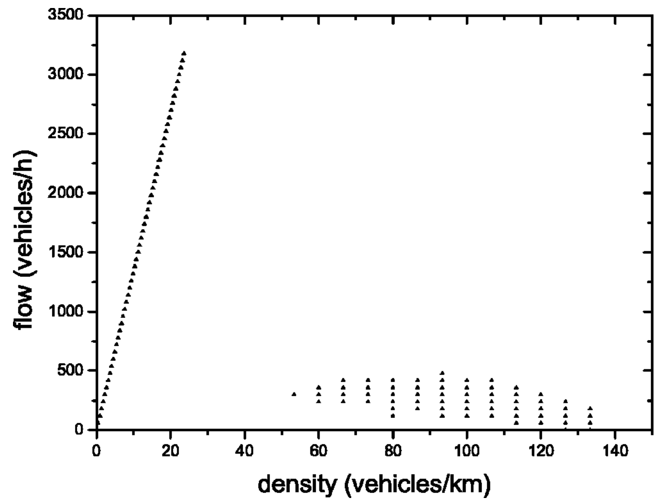

FIG. 7. The 1-min average data of the present model. Only the data inside free flow and wide jam are recorded. The data in free flow are calculated by $k=J/v$. The density in jams is calculated as shown in footnote 3, the flux in jams is obtained by recording the number of vehicles passing x in the time interval $(t-30,t+30)$ s.

One can see that the system is also the coexistence of free flow and jam. But the jam in the present model is explicitly different from that in the VE-STS model, it is more sparse and the blanks inside are visible. This is consistent with the experimental observations.

With the increase of the global density, the free flow region shrinks and the jam region expands, see Fig. 4(b). When the density exceeds the critical density k_{c5}, the free flow region will disappear and only jam will exist, see Fig. 4(c). If the density still increases, the blanks inside the sparse jam are exhausted gradually by the added vehicles. As a result, the density of the sparse jam will gradually increase and finally will turn into a completely compact jam at the density $k=1$.

The result of distribution of time headways in free flow region is the same as that in the VE-STS model because the concept of the ‘‘jammed status’’ only works in high density region.

For better understanding of the jams in the model, we present the simulation results of time series of the density at a fixed location inside the jams in Fig. 5. It can be seen that the density inside the jams fluctuates in a relatively wide range,[3] which is in accordance with the empirical data (cf. Fig. 23, Ref. [3]).

To distinguish the sparse jam and compact jam in the model, we show the evolution of the traffic behind a traffic light, see Fig. 6. It can be seen that there is a gradual transition from sparse jam to compact jam. Far away from the light, the jam is sparse in that drivers who notice there is a

[2]When using larger P_0 and P such as $P_0=0.7$ and $P=0.5$, the jams are not entirely compact. However, such larger randomization probability will lead to the unrealistic upstream moving velocity and the unrealistic outflow from jams. Therefore, such parameter regime is not investigated in this paper.

[3]Traditionally, the density is calculated by $k=J/v$, where J is the mean flux and v is the mean velocity of cars passing the detector in a time interval of 1 min. Nevertheless, this method always underestimates the density. Here the density in the jams is calculated by averaging over a small stretch of length $L_x=100$ between $x-L_x/2$ and $x+L_x/2$ at time t.

red light forwardly will brake in advance and move ahead step by step. So there will be large gaps between those vehicles. Near the light, the vehicles are bumper to bumper because the first car behind the light is halted all the while and the blanks behind it are reduced by the vehicles approaching it. The longer the time when the light keeps red, the wider the compact jam region. This result is in accordance with the situation behind a light in real traffic.

Finally we show the 1-min. average data of the present model in Fig. 7. It can be seen that the jams are identified by a triangular structure at large densities and small flows. This also implies that the jams are not compact and the gaps inside lead to small values of flow, which is qualitatively consistent with the empirical data (cf. Fig. 22, Ref. [3]).

III. CONCLUSIONS

In this paper we present a CA model to simulate the complex structure of wide jams. By introducing ''jam headway'' and ''jammed status'' on the basis of VE-STS model, we take into account the different behaviors the drivers adopt when they enter jammed region and free flow region. The model can represent wide sparse jams with gaps inside it as well as the platoon of vehicles with high speed in free flow. The fundamental diagram is studied in details. It is shown that when the density is in the intermediate range, there will be no compact jams, instead, the system is a coexistence of free flow and sparse jam. The simulation result of the jam formation behind a traffic light, which shows the transition from sparse jam to compact jam, illuminates the relation between these two states. The time series of the density inside wide jams and the 1-min average results of density-flow relation accord with the real traffic measurements.

Recently, the analysis of lots of experimental measurements indicates that the traffic behaviors not only depend on the local density but also on the traffic state [3]. For example, in jammed region, the vehicles are slower than the gaps allow, while in free flow, the velocity shows large values even for small headway. Therefore, to investigate different reactions of the drivers in different traffic states is very important to the in-depth development of traffic theory. In this paper, we proposed concepts of ''jam headway'' and ''jammed status'' in order to model the special behavior of drivers in congested flow. Together with the thought of velocity effect model that takes into account the movement of proceeding cars, especially in free flow, we get the detailed structures of wide jam and free flow as expected, which verifies the feasibility of considering different behaviors under different traffic states. Nevertheless, as NS model and STS model, this model does not reproduce the phenomena of synchronized traffic. In our future work, we will try to include the synchronized flow into this model and thoroughly study the effects of different reactions that drivers take in free flow, synchronized flow, and wide jams.

ACKNOWLEDGMENT

This work was financially supported by the National Natural Science Foundation of China with Grant No. 10272101.

[1] L. Neubert, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **60**, 6480 (1999).

[2] B. Tilch and D. Helbing, in *Traffic and Granular Flow '99*, edited by D. Helbing, H.J. Hermann, M. Schreckenberg, and D.E. Wolf (Springer, Heidelberg, 2000), p. 333.

[3] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **65**, 056133 (2002).

[4] B.S. Kerner and H. Rehborn, Phys. Rev. E **53**, 1297 (1996).

[5] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001).

[6] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).

[7] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).

[8] A. Schadschneider and M. Schreckenberg, Ann. Phys. (Paris) **6**, 541 (1997).

[9] R. Barlovic, L. Santen, A. Schadschneider, and M. Schreckenberg, Eur. Phys. J. B **5**, 793 (1998).

[10] X.B. Li, Q.S. Wu, and R. Jiang, Phys. Rev. E **64**, 066128 (2001).

[11] B.S. Kerner, Phys. Rev. Lett. **81**, 3797 (1998).